## Bounds and Error Intervals (Rounding)



A number has been rounded to the nearest whole number. In this case, that falls as 37 .

What is the lower bound and what is the upper bound?
The lower bound is the lowest number that would be rounded up to 37 . In this case, that would be 36.5 as any less would be rounded down to 36 .

The upper bound is the highest number that would be rounded down to 37. In this case, that would be 37.4999999999999999999999999999999999999999999999999... infinite number of 9s ... 9999 .

For convenience, there is an agreement that we state this as 37.5 . So the upper bound is 37.5 .

1
The following have been rounded to one decimal place. What are the lower and upper bounds?


2 The following have been rounded to two decimal places. What are the lower and upper bounds?


3 The following have been rounded to three decimal places. What are the lower and upper bounds?


4 Give the upper and lower bounds of the following rounded numbers to an appropriate number of decimal places.


5 Bill measures the length of his boat and rounds the length to the nearest metre. It is 12 metres long. What are the upper and lower bounds of the length?
6 Cinnamon is baking buns. She rounds the number of buns to the nearest 10. She has baked about 60 buns. What are the lower and upper bounds of the number of buns she has baked?
$7 \quad$ Freda runs a mile in 7 minutes to the nearest minute. What are the lower and upper bounds of how fast she has run the mile?


The length of a boat is 25 m rounded to the nearest metre.
The error interval is the lower bound $\leq$ length $<$ upper bound. Note that the signs are different to account for the fact that when we round, 37.5 would round up to 38 .

So the error interval would be $24.5 \leq$ length $<25.5$.
The error interval is the limit of the possible values of a particular variable that has been rounded or truncated. The length of the boat could be anything in between 24.5 m and 25.5 m so it could be 24 m 87 cm or 25 m 49.8 cm . We don't know the actual value but we do know it is between the upper and lower bounds.
8. Work out and write the error intervals for the following.
a A car is measured to the nearest 100 kg . It has a mass of 900 kg .
b A door height is measured to the nearest mm . It is measured as 1950 mm .
c The width of a plate has been measured to the nearest cm . It is 23 cm in diameter.
9. The following have been rounded to one significant figure. Write the error interval for each. The type of measure is given in parentheses.
i) 600 miles (distance)
v) 800 yards (length)
ii) 5,000 tonnes (weight)
vi) 20 seconds (time)
iii) 0.007 mm (length)
vii) $70 \mathrm{kgm}^{-3}$ (density)
iv) 30 metres (length)
viii) $90 \mathrm{kgm}^{-2}$ (pressure)

10 The dimensions of the right-angled triangle are shown below rounded to two significant figures.


a Write the error interval of the area of the oblong.
b Write the error interval of the perimeter of the oblong.

12 A circular disc is 2 mm deep rounded to one significant figure. The disc has a diameter of 3 metres, again, rounded to one significant figure. The weight of the disc is 40 kg rounded to one significant figure. What is the error interval of the density of the disc?

Help: density, $\rho=\frac{\text { mass }}{\text { volume }}$ and is measured in $\mathrm{kgm}^{-3}$

## Suitable Degrees of Accuracy

Sometimes you will be asked to give your answer to a suitable degree of accuracy. This means that you need to give your answer to the number of significant figures where the lower and upper bound match.

For example, if you had an upper bound of 7.21673837 and a lower bound of 7.21755847 then you would give the answer of 7.21 as after this point, the two numbers diverge from each other.

13 Give the following to a suitable degree of accuracy.
a $\quad 83.399$ and 84.019
b $\quad 943.89288282$ and 943.8998321
c $\quad 5738.938484$ and 5993.938293
d $\quad 1234.6789$ and 1234.5678

14 The length of the edges of a cube are 9 cm long, rounded to one significant figure. The mass of the cube is 0.5 kg when rounded to one significant figure. Give the density of the cube to a suitable degree of accuracy.

Help: density, $\rho=\frac{\text { mass }}{\text { volume }}$ and is measured in $\mathrm{kgm}^{-3}$

