## **Bounds and Error Intervals (Rounding)**



A number has been rounded to the nearest whole number. In this case, that falls as 37.

What is the lower bound and what is the upper bound?

The lower bound is the lowest number that would be rounded up to 37. In this case, that would be 36.5 as any less would be rounded down to 36.

For convenience, there is an agreement that we state this as 37.5. So the upper bound is 37.5.

1 The following have been rounded to one decimal place. What are the lower and upper bounds?

8.5		12.7	 2.6	1	37.0	1
	J	<b>I</b>				

2 The following have been rounded to two decimal places. What are the lower and upper bounds?

1.76	J	23.	86	942	2.18		0.	70	
	-				-	_			_

3 The following have been rounded to three decimal places. What are the lower and upper bounds?

8.754	12.423	0.067	-3.467

4 Give the upper and lower bounds of the following rounded numbers to an appropriate number of decimal places.

16	23.85	-7	-5.4
	] []	J LL	

- 5 Bill measures the length of his boat and rounds the length to the nearest metre. It is 12 metres long. What are the upper and lower bounds of the length?
- 6 Cinnamon is baking buns. She rounds the number of buns to the nearest 10. She has baked about 60 buns. What are the lower and upper bounds of the number of buns she has baked?
- 7 Freda runs a mile in 7 minutes to the nearest minute. What are the lower and upper bounds of how fast she has run the mile?



The length of a boat is 25m rounded to the nearest metre.

The error interval is the *lower bound*  $\leq$  *length* < *upper bound*. Note that the signs are different to account for the fact that when we round, 37.5 would round up to 38.

So the error interval would be  $24.5 \leq length < 25.5$ .

The error interval is the limit of the possible values of a particular variable that has been rounded or truncated. The length of the boat could be anything in between 24.5m and 25.5m so it could be 24m 87cm or 25m 49.8 cm. We don't know the actual value but we do know it is between the upper and lower bounds.

- 8. Work out and write the error intervals for the following.
  - a A car is measured to the nearest 100 kg. It has a mass of 900 kg.
  - b A door height is measured to the nearest mm. It is measured as 1950mm.
  - c The width of a plate has been measured to the nearest cm. It is 23cm in diameter.
- 9. The following have been rounded to one significant figure. Write the error interval for each. The type of measure is given in parentheses.
  - i) 600 miles (distance)
  - ii) 5,000 tonnes (weight)
  - iii) 0.007 mm (length)
  - iv) 30 metres (length)

- v) 800 yards (length)
- vi) 20 seconds (time)
- vii) 70kgm<sup>-3</sup> (density)
- viii) 90 kgm<sup>-2</sup> (pressure)

10 The dimensions of the right-angled triangle are shown below rounded to two significant figures.



- Find the upper and lower bounds of the area of the triangle.
- What is the percentage difference between the lower and upper bounds?
  - Write out the error interval for the area of the triangle.

11 The dimensions of the oblong below are rounded to 2 significant figures.



12 A circular disc is 2mm deep rounded to one significant figure. The disc has a diameter of 3 metres, again, rounded to one significant figure. The weight of the disc is 40kg rounded to one significant figure. What is the error interval of the density of the disc?

Help:  $density, \rho = \frac{mass}{volume}$  and is measured in kgm<sup>-3</sup>

## **Suitable Degrees of Accuracy**

Sometimes you will be asked to give your answer to a suitable degree of accuracy. This means that you need to give your answer to the number of significant figures where the lower and upper bound match.

For example, if you had an upper bound of 7.21673837 and a lower bound of 7.21755847 then you would give the answer of 7.21 as after this point, the two numbers diverge from each other.

- 13 Give the following to a suitable degree of accuracy.
  - a 83.399 and 84.019
  - b 943.89288282 and 943.8998321
  - c 5738.938484 and 5993.938293
  - d 1234.6789 and 1234.5678
- 14 The length of the edges of a cube are 9cm long, rounded to one significant figure. The mass of the cube is 0.5 kg when rounded to one significant figure. Give the density of the cube to a suitable degree of accuracy.

Help: density,  $\rho = \frac{mass}{volume}$  and is measured in kgm<sup>-3</sup>